Internal Assessment Resource

Achievement Standard Mathematics and Statistics 91574: Apply linear programming methods in solving problems

Resource reference: Mathematics and Statistics 3.2A

Resource title: Ted's tomatoes

Credits: 3

Achievement	Achievement with Merit	Achievement with Excellence
Apply linear programming methods in solving problems.	Apply linear programming methods, using relational thinking, in solving problems.	Apply linear programming methods, using extended abstract thinking, in solving problems.

Student instructions

Introduction

Ted grows artichokes and tomatoes. He is looking at planting options to maximise his income.

This activity requires you to use linear programming to model the constraints Ted has for his planting and to make recommendations so that he can maximise his income in the current year and in future years. You will present your findings as a written report, supported by graphs, equations and relevant calculations.

The quality of your reasoning and how well you link this to the context will determine the overall grade.

You have <<teacher to insert time here>> in which to independently complete this task.

Task

Ted produces tomatoes for a local factory and sells artichokes at a farmers' market. Artichokes are very labour-intensive and Ted is looking at his planting options in order to maximise his income.

Using the constraints outlined in the Resource Sheet, write a report making recommendations as to how many hectares of tomatoes and artichokes Ted should plant to maximise his income in the current year and in future years.

As you write your report take care to clearly communicate your findings using appropriate mathematical statements. Include graphs, equations, and relevant calculations.

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Resource Sheet

Constraints

Artichokes require 20 hours of labour per hectare, tomatoes require 10 hours per hectare. Ted has 1200 hours of labour available for the two crops.

Ted has 90 hectares available for planting. This can be written as $t + a \leq 90$, where t is the number of hectares planted in tomatoes and a is the number of hectares planted in artichokes.

To keep his contract with the factory, Ted must plant at least 30 hectares of tomatoes. He feels that planting anything less than 10 hectares of artichokes would make travel to the farmers' market uneconomical.

Income predictions

For the current year, Ted expects to receive $10,000 per hectare for his tomatoes and $25,000 per hectare for his artichokes. His income, I, can be expressed as $I = 10\,000t + 25\,000a$.

The future value of tomatoes and artichokes is unknown. However, payments per hectare of tomatoes and hectare of artichokes are forecast to be in a ratio of 1:2.

- Let a = number of hectares of artichokes
  t = number of hectares of tomatoes.

equations
- Labour: $10t + 20a \geq 1200$
- Space: $t + a \leq 90$
- contract: $t \geq 30$
- economy: $a \geq 10$

Graph.

coordinates

$a = (30, 10)$ $b = (30, 45)$
$c = (60, 30)$ $d = (80, 10)$

Income at each Point

$I = 10\,000t + 25000a$

$a = 10000 \times 30 + 25000 \times 10$
$I = \$550,000$

$b = 10000 \times 30 + 25000 \times 45$
$I = \$1,145,000$

$c = 10\,000 \times 60 + 25\,000 \times 30$
$I = \$1,350,000$

$d = 10\,000 \times 80 + 25000 \times 10$
$I = \$1,050,000.$

Maximise

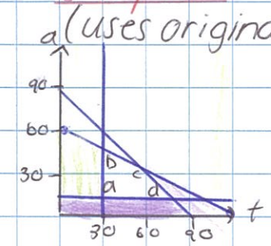
From the coordinates of the optimal points, I can determine that the maximum income Ted can make (given the restraints) is $1,145,000. Therefor in order to generate a maximum profit, Ted should plant 30 hectares of tomatoes and 45 hectares of artichokes.

Change to income equation

equation

$I = t + 2a$

Graph. (uses original)



Coordinates

$a = (30, 10)$
$b = (30, 45)$
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Future income at each Point

$I = t + 2a$

$a = 30 + (2 \times 10)$
$I = \$50$

$b = 30 + (2 \times 45)$
$I = \$120$

$c = 60 + (2 \times 30)$
$I = \$120$

$d = 80 + (2 \times 10)$
$I = \$100$

Gradient

$m = -\frac{a}{b} =$

- $I = t + 2a$
  $m = \frac{-1}{2}$

- $10t + 20a \geq 12000$
  $m = \frac{-10}{20}$ or $\frac{-1}{2}$

Maximise

Given the new, predicted ratio, the equation for income changes. This new ratio is modelled by: $I = t + 2a$. This means that the points at which Ted maximises his profit will also change. As both equations $I = t + 2a$ and $10t + 20a \geq 12000$ have the same gradient of $-\frac{1}{2}$, there will be multiple solutions along the line of $10t + 20a \geq 12000$ between 30 and 60 hectares of tomatoes and 30 and 45 hectares of artichokes in order to get maximum profit in future. An example of use of planting of space that Ted could use to get optimum income is if he planted 45 hectares of tomatoes and 40 hectares of artichokes which would give $(100 \times 45 + 2 \times 100 \times 40) = \$12500$ return - if the

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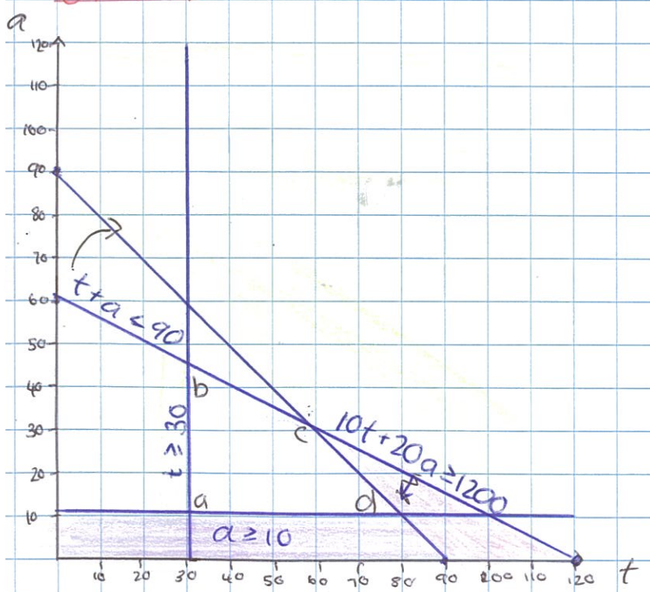

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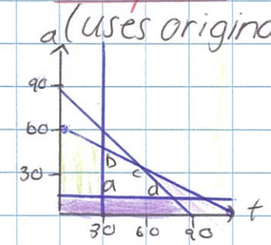

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