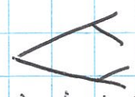
TOPIC PROBABILITY A.S. 3.13 4 CREDITS EXTERNAL OVERVIEW

- basic probability - as per year 12
- venn diagrams (A ∩ B)
- two way tables

	A	A'
C		
C'		

⇒ in/ε comes from doing comparisons similar to yr 12 relative risk

- probability trees



- risk / rel risk
- Simulations, experimental, theoretical, true probability
- probability tables and expected value

for any of these you may need:

- union: P(A∪B)
- intersection: P(A∩B)
- complement: P(A')
- mutally exclusive ⟺ P(A∩B) = 0
- independant ⟺ P(A∩B) = P(A) × P(B)
- conditional $P(A|B) = \frac{P(A \cap B)}{P(B)}$

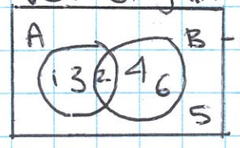
⟺ = iff

means relationship goes both ways

SET NOTATION

eg $A = \{1, 2, 3\}$ $B = \{2, 4, 6\}$

venn diagram



- universal set of counting numbers to six

★ "∈" means "is an element of" = a member of

eg $2 \in A$ and B

$3 \in A$ $3 \notin B$

★ "∪" means "Union" or "Or"

A∪B means everything in set A or set B

eg $A \cup B = \{1, 2, 3, 4, 6\}$

★ "∩" means "intersection", "overlap" = everything which is in both A and B.

eg $A \cap B = \{2\}$

★ "A'" (Or A^c in america) means "the complement of A" or everything not in A or not A for short

eg $A' = \{4, 5, 6\}$

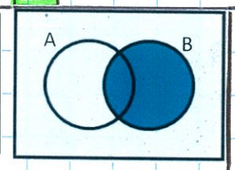
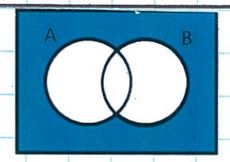
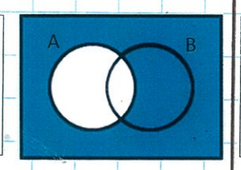
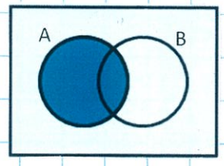
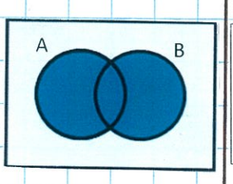
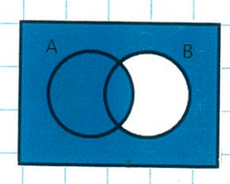
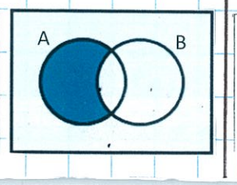
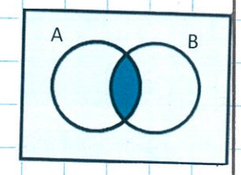
$B' = \{1, 3, 5\}$

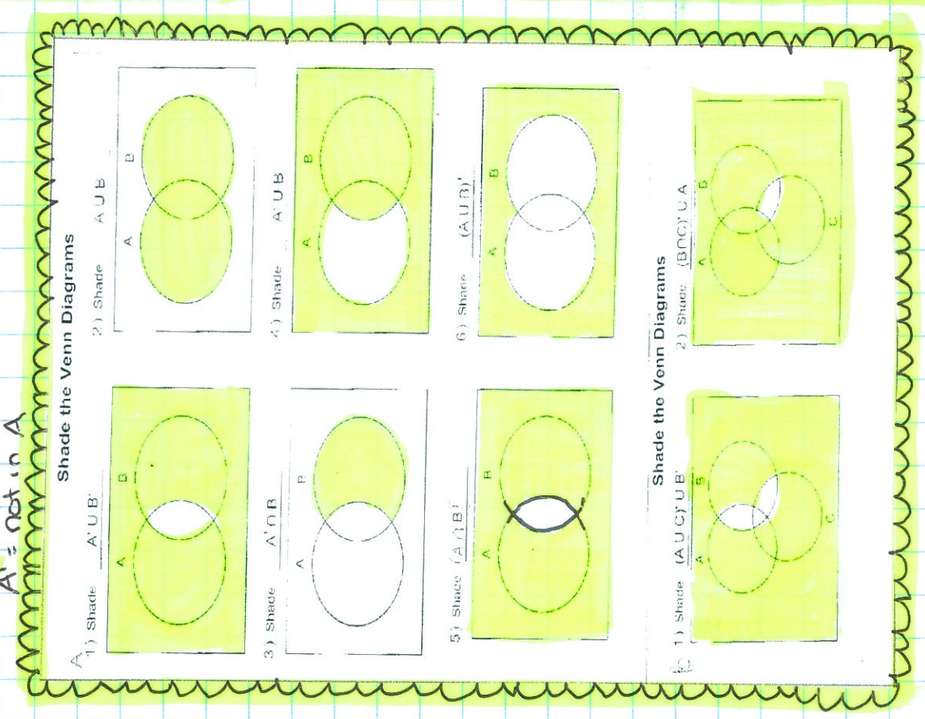
$(A \cup B)' = \{5\}$

$(A \cap B)' = \{1, 3, 4, 5, 6\}$

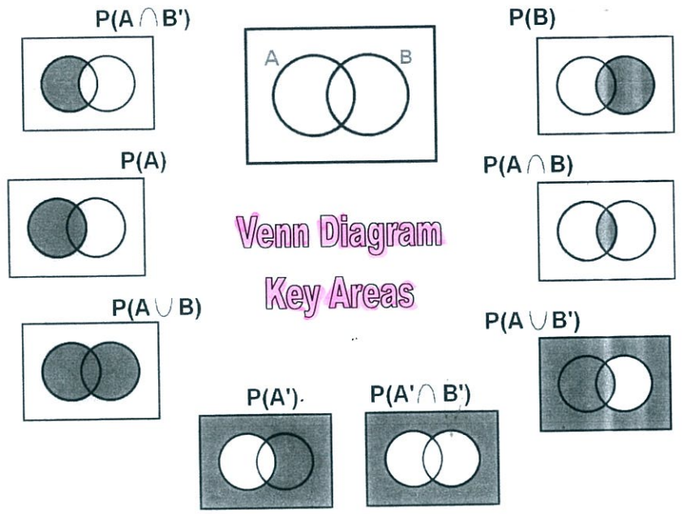
$A' \cup B = \{4, 5, 6\}$

$A \cap B' = \{1, 3\}$

B		A'∩B'	
A'		A	
A∪B		A∪B'	
A∩B'		A∩B	



Union = "or" +
∩ Intersection = both & overlap
A' = not in A

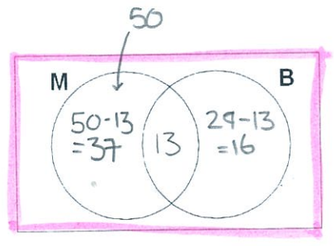


P(A) + P(A') = 1

Question:

In an group of 100 students:
50 study Mathematics, 29 study Biology, 13 study both.

Draw a Venn Diagram and use it to find:



P(M) = $\frac{50}{100}$

P(B) = $\frac{29}{100}$

P(M ∪ B) = $\frac{66}{100}$

P(M ∩ B) = $\frac{13}{100}$

P(M') = $\frac{16+34}{100} = \frac{50}{100}$

P(B') = $\frac{100-29}{100}$ or $\frac{34+37}{100} = \frac{71}{100}$

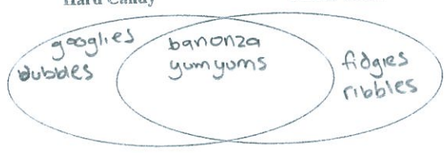
P(M ∩ B') = $\frac{37}{100}$

Solve each problem.

1) The table below shows the attributes of several types of candy. Use the chart to fill in the Venn Diagram.

	Banonza	Googlies	Dubbles	Yum-Yums	Fidgies	Ribbles
Type	Hard	Hard	Hard	Hard	Soft	Soft
Color	Mixed	Uniform	Uniform	Mixed	Mixed	Mixed

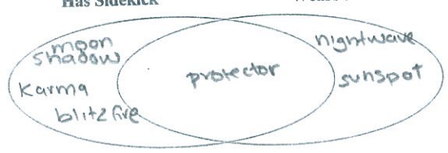
Hard Candy **Mixed Color**



2) The table below shows the description of several super heroes. Use the chart to fill in the Venn Diagram.

	Moon Shadow	Karma	Blitz Fire	Nightwave	Sunspot	Protector
Has Sidekick	Yes	Yes	Yes	No	No	Yes
Wears Mask	No	No	No	Yes	Yes	Yes

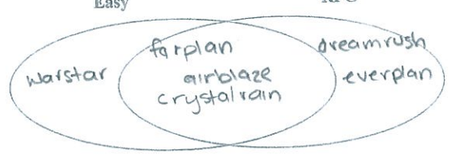
Has Sidekick **Wears Mask**



3) The table below shows the description of several video games. Use the chart to fill in the Venn Diagram.

	Farplan	DreamRush	Everplan	Airblaze	Crystal Rain	Warstar
Difficulty	Easy	Hard	Hard	Easy	Easy	Easy
Type	RPG	RPG	RPG	RPG	RPG	Action

Easy **RPG**

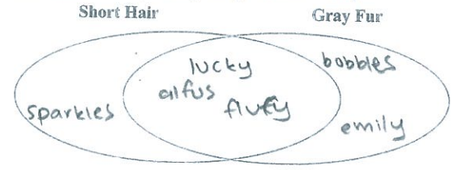


Solve each problem.

1) The table below shows the physical traits of several pets in a pet shelter. Use the chart to fill in the Venn Dia

	Bubbles	Lucky	Alfus	Fluffy	Sparkles	Emily
Fur Length	Long	Short	Short	Short	Short	Long
Fur Color	Gray	Gray	Gray	Gray	Brown	Gray

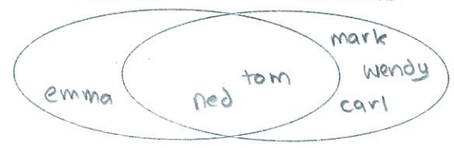
Short Hair **Gray Fur**



2) The table below shows the physical traits of different students. Use the chart to fill in the Venn Diagram.

	Mark	Emma	Ned	Tom	Carl	Wendy
Hair Color	Red	Blonde	Blonde	Blonde	Red	Red
Dominant Hand	Left	Right	Left	Left	Left	Left

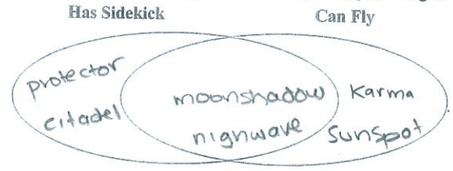
Blonde Hair **Left Handed**



3) The table below shows the description of several super heroes. Use the chart to fill in the Venn Diagram.

	Protector	Moon Shadow	Karma	Sunspot	Nightwave	Citadel
Has Sidekick	Yes	Yes	No	No	Yes	Yes
Power	Super Strength	Flight	Flight	Flight	Flight	Super Strength

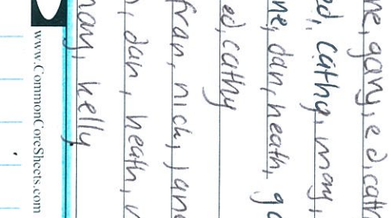
Has Sidekick **Can Fly**



TYPES of PROBABILITY:

- theoretical Probability - you can calculate using rules (P99)
- experimental Probability - using results of an experiment
- true/real Probability - rarely actually know - the actual probability of an event occuring in the real world.

The diagram below shows the different transportation students had. Bike (B), Scooter (S) and Roller Blades(R). Use the diagram to answer the questions.



1) How many people had a bike? 8
2) How many people had a scooter? 6
3) How many people had roller blades? 6
4) How many people had ONLY a bike? 2
5) How many people had ONLY a scooter? 1
6) How many people had ONLY roller blades? 1
7) S∪B = anne, gary, ed, cathy, dan, heath, mary, kelly
8) B∩R = ed, cathy, mary, kelly
9) B-R = anne, dan, heath, gary
10) (R∩B)-S = ed, cathy
11) (S∪R)-B = fran, nick, jane
12) S - fran, dan, heath, mary, kelly, nick.
13) BRS = mary, kelly

Answers

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. _____ (inc)
8. _____ (inc)
9. _____ (inc)
10. _____ (inc)
11. _____ (inc)
12. _____ (inc)
13. _____ (inc)

1-10	92	85	77	69	62	54	46	38	31	23
11-13	15	8	0							

eg favfood

DISCRETE
- things that you measure

DATA

CONTINUOUS
- things that you measure eg height

DISCRETE DATA SETS

p6

$P(X>3)$ 0 1 2 3 4 5 6 7 8 $P(X \geq 1)$

$P(X \leq 2)$ 0 1 2 3 4 5 6 7 8 $P(X<3)$

$P(2<X<7)$ 0 1 2 3 4 5 6 7 8 $P(3 \leq X \leq 6)$

$P(X \geq 4)$ 0 1 2 3 4 5 6 7 8 $P(X > \quad)$

p7

P(X is exactly 4) 0 1 2 3 4 5 6 7 8 $P(X=4)$

P(X is greater than 2) 0 1 2 3 4 5 6 7 8 $P(X>2)$ or $P(X \geq 3)$

P(X is between 3 & 7) 0 1 2 3 4 5 6 7 8 $P(3<X<7)$ or $P(4 \leq X \leq 6)$

If Mr Dean walked in to the room now and chose two students at random to help him, how many different ways could he make this choice?

What if he chose 3 students?

The Multiplication principle:

If A can be done in n ways and B can be done in m ways then A followed by B can be done in $n \times m$ ways.

Eg.1 How many outfits (of shoes, skirt and top) does a female year 13 student have for school if she can choose from 2 pairs of shoes, 4 skirts and 5 tops?

Eg.2 There are 8 doors in a hall. In how many different ways can they be left either open or closed?

Eg.3 In the UK, number plates have 2 letters, followed by 2 digits, then 3 more letters; eg. SW43QPA. The first digit cannot be 0. How many possible number plates are there?

eg1

shoes skirt tops

$2 \times 4 \times 5 = 40$

eg2

D1 D2 D3 D4 D5 D6 D7 D8

$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 8 = 256$

$2^8 = 256$

eg3

L1 L2 N1 N2 L3 L4 L5

$26 \times 26 \times 9 \times 10 \times 26 \times 22 \times 26 =$

1,069,323,840

FINDING PROBABILITIES USING THE MULTIPLICATION PRINCIPLE

work out:

how many ways there are to choose each targeted item the total remaining number of items

for each step, then multiply these together

eg if Mr Dean comes in to choose 3 students from our class of 18, the probability he chooses Amy, Cara and Mana (if in any order)

ans $\frac{13}{18} \times \frac{?}{17} \times \frac{1}{16}$

ways to choose first person

FACTORIALS

if n is a whole number $[W = \{0, 1, 2, 3 \cdots\}]$ then n factorial (written n!) is the product of all whole numbers from 1 to n.

eg $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$ $\frac{6!}{6} = 5!$ $\frac{5!}{5} = 4!$

$4! = 4 \times 3 \times 2 \times 1 = 24$

$3! = 3 \times 2 \times 1 = 6$

$2! = 2 \times 1 = 2$

$1! = 1$

$0! = 1$ $\left[\frac{1!}{1} = \frac{1}{1} = 1\right]$

n has been defined to be 1

OPTN → ▷ → PROB → 5c!

eg 8 students are waiting to use a single toilet. How many ways could they enter?

ans $8 \times 7 \times 6 \times \ldots \times 2 \times 1$

$= 8!$

$= 40,320$

PROBABILITY TABLES

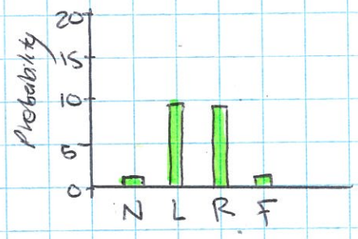
frequencies

no licence	learners	restricted	full	
1	9	9	1	= 20

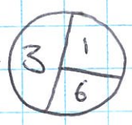
probabilities distribution table

no licence	learners	restricted	full	
1/20 = 0.05	9/20 = 0.45	9/20 = 0.45	1/20 = 0.05	20/20 = 1

WHAT PROPORTION OF STUDENTS HAVE A LICENCE?



expected number



x	1	3	6	Total
P(X=x)	1/4	2/4	1/4	4/4 = 1

what is the expected value or average?
1+3+3+6 = 13 13÷4 = 3.25

$\mu = E(X) = \sum x \cdot p(x)$
mean = expected value/No.; $\sum$ = sum of

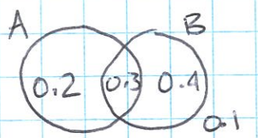
$E(X) = 1 \times \frac{1}{4} + 3 \times \frac{2}{4} + 6 \times \frac{1}{4} = 3.25$

Quick Exercise

copy & complete the probability distribution table and find the expected value

x	1	2	4	total
p(x)	1/8	3/8		

E(X) =



P(A) = 0.2, 0.3 = 0.5
P(B) = 0.3, 0.4 = 0.7
P(A∪B) = 0.2, 0.3, 0.4 = 0.9
P(A∩B) = 0.3
P(A∪B)' = P(A'∩B') = 0.1

RULES TO REMEMBER

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ - union

$P(A') = 1 - P(A)$ $P(A) + P(A') = 1$
- complement

if two events A and B are independent $\iff$ then $P(A) \times P(B) = P(A \cap B)$

PROBABILITY RULES

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$P(A') = 1 - P(A)$
$P(A \cap B) = P(A) \times P(B)$

notes: conditional Probability (given)

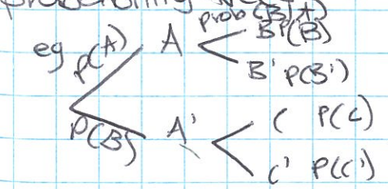
- they happen when one condition is already known

*this may mean that your probability is "out of" a particular row or column in a table/venn diagram

*or, it may mean you start part way in on a probability tree

*if you cant use these methods you may need the conditional formula: the probability of A "given B (has happened)" is: $P(A|B) = \frac{P(A \cap B)}{P(B)}$

NB CONDITIONAL PROBABILITIES MAY also need to be complete probability trees



eg. pg 45